$$= \lim_{x\to 1} \frac{(x+2)-(2x+1)}{(x-1)(\sqrt{x+2}+\sqrt{2x+1})} = \lim_{x\to 1} \frac{-(x-1)}{(x-1)(\sqrt{x+2}+\sqrt{2x+1})}$$

$$= \lim_{x\to 1} \frac{-1}{\sqrt{x+2}+\sqrt{2x+1}} = \frac{-1}{\sqrt{3}+\sqrt{3}} = \frac{-1}{2\sqrt{3}}.$$

3°/ $$\lim_{x\to 1} \frac{\sqrt{x^3+1}-\sqrt{x^3+x^2}}{x-1} = \lim_{x\to 1} \frac{(\sqrt{x^3+1}-\sqrt{x^3+x^2})(\sqrt{x^3+1}+\sqrt{x^3+2})}{(x-1)(\sqrt{x^3+1}+\sqrt{x^3+x^2})}$$

$$= \lim_{x\to 1} \frac{1-x^2}{(x-1)(\sqrt{x^3+1}+\sqrt{x^3+x^2})} = \lim_{x\to 1} \frac{-(x-1)(x+1)}{(x-1)(\sqrt{x^3+1}+\sqrt{x^3+x^2})}$$

$$= \lim_{x\to 1} \frac{-(x+1)}{\sqrt{x^3+1}+\sqrt{x^3+x^2}} = \frac{-\sqrt{2}}{2}$$

4°/ • On a : $x^2-x-2=(x+1)(x-2)$ et lorsque $x\to 2^+$,

$|x-2|=x-2$ et $|x|=x$ donc

$$\lim_{x\to 2^+} \frac{|x|\cdot|x-2|}{x(x^2-x-2)} = \lim_{x\to 2^+} \frac{x(x-2)}{x(x+1)(x-2)} = \lim_{x\to 2^+} \frac{1}{x+1} = \frac{1}{3}.$$

Lorsque $x\to 2^-$, $|x-2|=-(x-2)$ et $|x|=x$ donc

$$\lim_{x\to 2^-} \frac{|x|\cdot|x-2|}{x(x^2-x-2)} = \lim_{x\to 2^-} \frac{-x(x-2)}{x(x+1)(x-2)} = \lim_{x\to 2^-} \frac{-1}{x+1} = \frac{-1}{3}.$$

on a : $\lim_{x\to 2^-} \frac{|x|\cdot|x-2|}{x(x^2-x-2)} \neq \lim_{x\to 2^+} \frac{|x|\cdot|x-2|}{x(x^2-x-2)}$ donc la fonction

$x\mapsto \frac{|x|\cdot|x-2|}{x(x^2-x-2)}$ n'admet pas de limite lorsque $x\to 2$.

5°/ $$\lim_{x\to 0} \frac{x^2+|x|}{x^2-|x|} = \lim_{x\to 0} \frac{|x^2|+|x|}{|x^2|-|x|} = \lim_{x\to 0} \frac{|x|(|x|+1)}{|x|\cdot(|x|-1)} = \lim_{x\to 0} \frac{|x|+1}{|x|-1} = -1.$$